WS #5-6

Phase Shifts and Sinusoidal Curve fitting

1. **General Sinusoidal Equations**

$y = A\sin(bx - c) + d$ $\qquad$ $y = A\cos(bx - c) + d$

$y = A\sin\left[b\left(x - \frac{c}{b}\right)\right] + d$ $\qquad$ $y = A\cos\left[b\left(x - \frac{c}{b}\right)\right] + d$

Amplitude = $|A|$

Period ~~Adjustment~~ = $\frac{2\pi}{b}$

Phase Shift = c/b

Vertical Shift = d

2. **The steps to graphing a sinusoidal function are:**

A. Find amplitude $|A|$, period $\left(\frac{2\pi}{b}\right)$, + increment $\left(\frac{\text{period}}{4}\right)$

B. Find starting point of one cycle of the graph [phase shift $= \frac{c}{b}$]

C. Convert your phase shift to a fraction that has the same denominator as your increment

D. Find your $x_1 - x_5$ values (5 keypoints)

E. Find your midline ($y = d$), max (amplitude + d), min (−Amp + d)

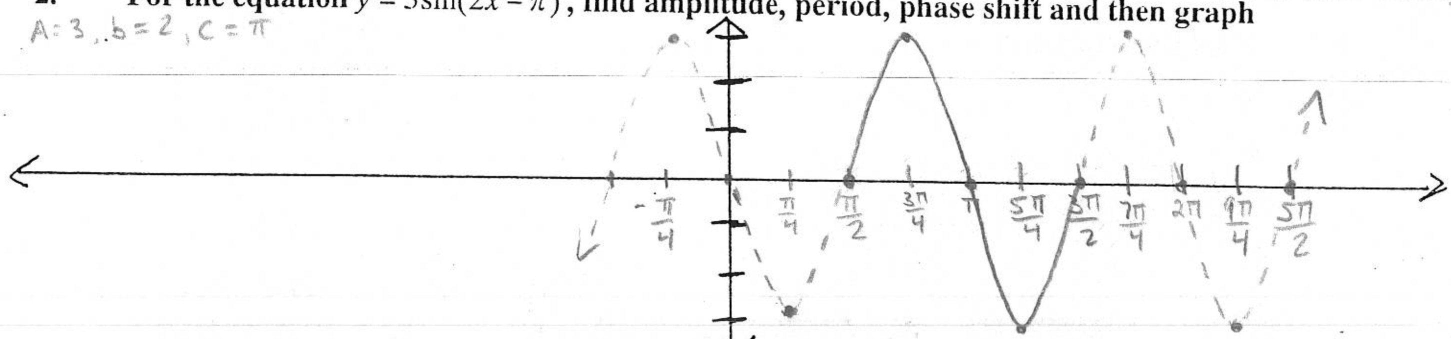
F. Scale your x-axis (according to 5 keypoints) + y-axis (according to max/min)

G. Plot points and connect w/ a smooth curve

- use 5 keypoints, max + min, $y = d$ (as x-int), + ±sin or ±cos function rules

2. For the equation $y = 3\sin(2x - \pi)$, find amplitude, period, phase shift and then graph

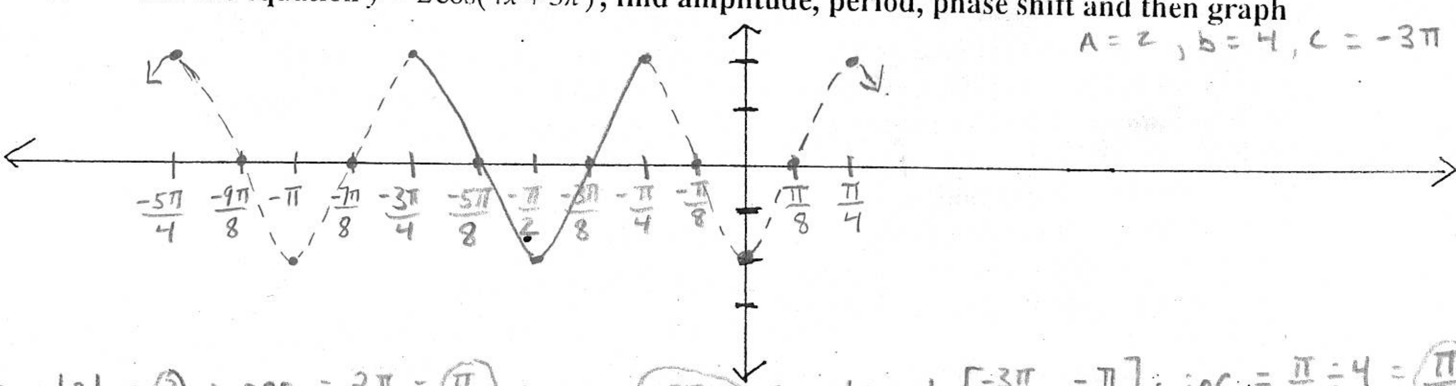
$A = 3,\ b = 2,\ c = \pi$



Amp $= |3| = 3$; per $= \frac{2\pi}{2} = \pi$; PS $= \frac{c}{b} = \frac{\pi}{2}$; interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$; inc $= \frac{\pi}{4}$

3. For the equation $y = 2\cos(4x + 3\pi)$, find amplitude, period, phase shift and then graph

$A = 2,\ b = 4,\ c = -3\pi$



Amp $= |2| = 2$; per $= \frac{2\pi}{4} = \frac{\pi}{2}$; PS $= -3\pi/4$; interval $\left[\frac{-3\pi}{4}, -\frac{\pi}{4}\right]$; inc $= \frac{\pi}{2} \div 4 = \frac{\pi}{8}$

$d = \frac{-1 + -5}{2} = -3$ per $= \frac{7\pi}{2} - \frac{-5\pi}{2} = \frac{12\pi}{2} = 6\pi$

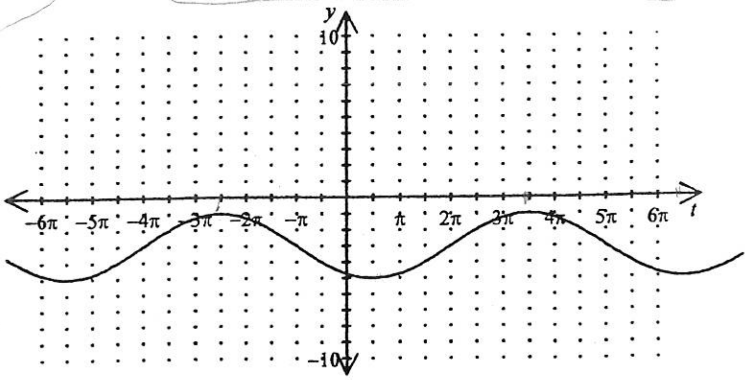
$b = 2\pi/6\pi = 1/3$ $C = \frac{1}{3} \cdot \frac{\pi}{2} = \frac{\pi}{6}$

$y = 3\sin(2x)$

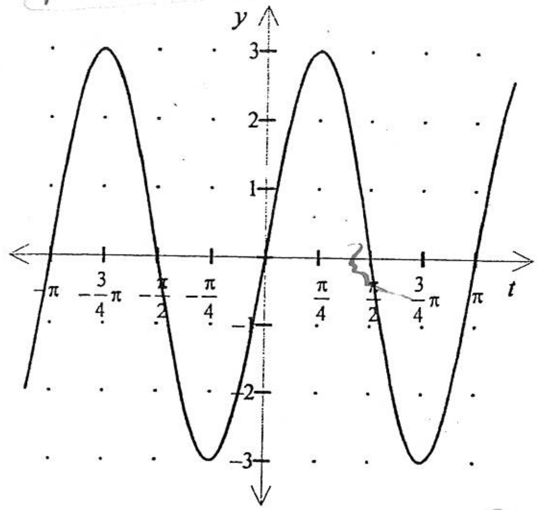
4. **Find the equation of the functions below:**

$A = \frac{-1 - -5}{2} = 2$, but A must be neg. if using cosine graph so $A = -2$

A. $\boxed{y = -2\cos\left(\frac{1}{3}x - \frac{\pi}{6}\right) - 3}$



B.



$b = 2\pi/\text{per} = 2\pi/\pi = 2$

5. **Data fitting**

Do All ?'s From #21 on p. 435 For this problem

Month, *x*	Average Monthly Temperature, °F
January, 1	29.7
February, 2	33.4
March, 3	39.0
April, 4	48.2
May, 5	57.2
June, 6	66.9
July, 7	73.5
August, 8	71.4
September, 9	62.3
October, 10	51.4
November, 11	39.0
December, 12	31.0

Source: U.S. National Oceanic and Atmospheric Administration

a.) Find sinusoidal function of best fit: put the #'s 1-12 in L_1 (for months of the yr) and the Avg monthly temperature's in L_2
↳ STAT : ENTER (then enter the above mentioned values in their lists)

b.) STAT → CALC : C (sin Reg) → Enter → Enter

c.) $y = 21.15\sin(0.55x - 2.35) + 51.19$

6. **Steps for fitting data to a sine or cosine function:**

A. Find A, the amplitude $= (\text{MAX} - \text{min}) \div 2$

B. Find d, the vertical shift $= (\text{MAX} + \text{min}) \div 2$

C. Find b, $b = 2\pi/\text{period}$

D. Find the horizontal shift (phase shift) and use

E. it + b to find C → $C = b \cdot \text{phase shift}$

$d: \frac{13.2 + 2.2}{2} = 7.7$ $b = \frac{2\pi}{\text{per}} = \frac{2\pi}{12.5 \text{ (length of time b/w consecutive high tides)}} = 2\pi \cdot \frac{2}{25} = \frac{4\pi}{25} = b$

$A = \frac{13.2 - 2.2}{2}$, $A = \frac{H}{2} = 5.5$

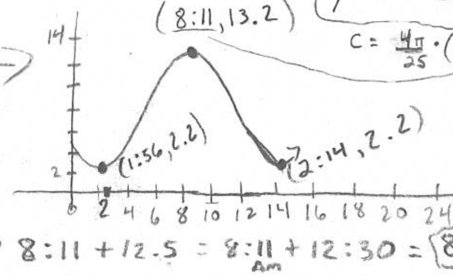
7. Suppose the length of time between consecutive high tides is approximately 12.5 hour. According to the National Oceanic and Atmospheric Administration, on Saturday, August 7, 2004, in Juneau, Alaska, a high tide occurred at 8:11 AM and a low tide occurred at 2:14 PM. (Water heights are measured as the amounts above or below the mean lower low tide.) The height of the water at high tide was 13.2 feet, and the height of the water at low tide was 2.2 feet.

$y = 5.5\sin\left(\frac{4\pi}{25}x - 2.5426\right) + 7.7$ x = time (as a decimal)

$y = 5.5\cos\left(\frac{4\pi}{25} - 4.1134\right) + 7.7$

$C = \frac{4\pi}{25} \cdot (8.1833) = 4.1134 = C$

A. Sketch the function



B. Find the function equations – sine and cosine

C. Approximate the time of the next high tide

$8{:}11 + 12.5 = 8{:}11 \text{ Am} + 12{:}30 = \boxed{8{:}41 \text{ pm}}$

1/2 of 12.5 hrs

D. What is the height of the tide at noon? $\boxed{5.8261 \text{ Ft}}$

8:11 am − 6:15 = 1:56 AM

PS = 1:56 + 8:11/2 = 10:07/2 = 5.0583 $C = b \cdot PS = \frac{4\pi}{25}(5.0583) = 2.5426 = C$